BA/ BSc 4th Semester

Roll No. ..... Total Pages : 04

GSQ/D-24 **952**

REAL ANALYSIS

BM-351

Time : Three Hours] [Maximum Marks : B. Sc. : 40, B. A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Let f be a bounded function on $[a, b]$. Then : **2,1**

$$\int_{\underline{a}}^{b} f\,dx \le \int_{a}^{\overline{b}} f\,dx$$

(b) Evaluate the improper integral $\int_1^2 \frac{x}{\sqrt{x-1}}dx$. **1½,1**

(c) Examine the convergence of integral $\int_0^{\pi/4} \frac{1}{\sqrt{\sin x}}dx$. **1½,1**

(d) Show that in discrete metric space X, every subset of X is open. **1½,1**

(e) Prove that isometry is a uniformly continuous function. **1½,1**

Unit I

2. (a) If f is a function defined on $[0, a]$; $a > 0$ by $f(x) = x^3$, then show that f is Riemann integrable on $[0, a]$ and $\int_0^a f\,dx = \frac{a^4}{4}$. 4,2½

(b) Prove that a continuous function f on $[a, b]$ is integrable on $[a, b]$. Is the converse true ? Justify. 4,2½

3. (a) State and prove fundamental theorem of integral calculus. 4,2½

(b) Show that : 4,2½

$$\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x^2}}\,dx \leq \frac{1}{3}$$

Unit II

4. (a) Examine the convergence of $\int_0^1 x^{n-1} \log x\,dx$. 4,3

(b) Test the convergence of $\int_0^\infty \left(\frac{1}{x} - \frac{1}{\sinh x}\right)\frac{1}{x}\,dx$. 4,3

5. (a) Examine the convergence of $\int_0^\infty \frac{\cos x}{\sqrt{x^2+x}}\,dx$. 4,3

(b) Evaluate $\int_0^{\infty} \frac{\tan^{-1} \alpha x}{x\left(1+x^2\right)} dx$ if $\alpha \geq 0$. 4,3

Unit III

6. (a) Let (X, d) be a metric space. Define $d^*: X \times X \to R$ by $d^*(x, y) = \min\{2, d(x, y)\}$. Show that (X, d^*) is a metric space. 4,3

(b) Prove that a subset A of a metric space is closed iff it contains each of its limit points. 4,3

7. (a) Prove that interior of a subset A of a metric space (X, d) is the largest open subset of A. 4,3

(b) Prove that a subspace Y of a complete metric space X is complete iff it is closed. 4,3

Unit IV

8. (a) Let (X, d) and (Y, d^*) be metric spaces and f be a function of X into Y. Then, f is continuous iff $f^{-1}(G)$ is open in X whenever G is open in Y. 4,2½

(b) A metric space is sequentially compact iff it has BWP. 4,2½

9. (a) A metric space (X, d) is compact iff every collection of closed subsets of X with finite intersection property has a non-empty intersection. **4,2½**

(b) Let (X, d) be a metric space and let $\{C_\lambda\}_{\lambda\in\Lambda}$ be a non-empty collection of connected subsets of X such that $\bigcap_{\lambda\in\Lambda}\{C_\lambda\}\neq\phi$. Then, $\bigcup_{\lambda\in\Lambda}\{C_\lambda\}$ is a connected set. **4,2½**

Bsc 5th Semester

Roll No. ..... Total Pages : 03

GSQ/D-24 957

QUANTUM AND LASER PHYSICS

Paper : IX

PH-502

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Differentiate between Group velocity and Phase velocity. **2**

(b) Calculate the least and maximum values of wavelength obtained when γ rays of energy 0.5 MeV are scattered by the matter. **2**

(c) Explain the concept of Quantum Mechanical Tunneling. **2**

(d) What is Line Broadening ? Elaborate. **2**

Unit I

2. (a) What will be the de-Broglie wavelength of an electron having kinetic energy of 700 eV. **2**

(b) Derive the expression for Kinetic energy of recoil electron in term of scattering angle of photons. **6**

3. (a) Explain the meaning of ψ and $|\psi|^2$. 2

(b) What is the importance of normalization of wavefunctions ? Show that two eigen functions corresponding to the different energy eigenvalues are orthogonal to each other. 6

Unit II

4. (a) Define step potential. Solve Schrödinger wave equation for a particle in step potential for $E > V_0$. 6

(b) Find the values of momentum and energy for an electron in a box of length 1 Å for $n = 1, 2$. 2

5. Solve Schrödinger wave equation to evaluate the energy levels for the ground state and excited states of Linear Harmonic Oscillator and show that its energy levels are discrete. 8

Unit III

6. (a) Write a note on types of coherence. For a source of light having coherence time $t_c \sim 10^{-9}$ s. Calculate the degree of non-monochromaticity for $\lambda_0 = 5800$ Å. 4

(b) Define Einstein's coefficient. Find the expression for ratio of spontaneous to stimulated emission in thermodynamic equilibrium. 4

7. (a) Derive the expression for Schawlow-Townes threshold condition for the laser oscillation. 6

(b) Explain the quantum amplification and quantum switch in active medium. 2

Unit IV

8. Discuss the principle, construction and working of Ruby laser with suitable diagrams. Out of He-Ne laser and Ruby laser, which has more monochromaticity ? 8

9. (a) What are the main features of semiconductor laser ? How does laser action take place in it ? 6

(b) Give some important applications of lasers in medicine and industry. 2

Roll No. .....

Total Pages : 2

B.Sc III V 758 Sem

GSE/D-24

ELECTRICITY, MAGNETISM & ELECTROMAGNETIC THEORY

Paper–II (PH–102)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question No. **1** is compulsory. All questions carry equal marks. Use of Scientific (Non–programmable) calculator is allowed.

Compulsory Question

1. Answer the following questions: 4×2=8

   (i) Show that an electrostatic field is curl free.

   (ii) Why soft iron is best suited for electromagnet?

   (iii) What are electromagnetic waves? Write the relation for speed of electromagnetic waves in vacuum.

   (iv) Write down the uses of series and parallel resonant circuits.

UNIT–I

2. (a) What is electric flux? State and prove Gauss's law in electrostatics. 4

   (b) Using the Gauss's law, derive the expression for the electric field intensity at appoint near an infinite plane sheet of charge. 4

3. (a) State and prove Gauss's divergence theorem. 4

(b) Derive Poisson's and Laplace's equations. 4

UNIT–II

4. (a) Explain the electronic theory of diamagnetism. 6

(b) What do you mean by Hysteresis? 2

5. (a) Prove that $\nabla.\vec{B} = 0$. 4

(b) Define magnetic induction, magnetic intensity, intensity of magnetizing and derive the relation between them. 4

UNIT–III

6. What is Poynting Vector? Give its physical significance and derive Poynting theorem. 8

7. (a) Prove that $\nabla.\vec{D} = \rho$. 4

(b) Define Scalar and Vector potentials. 4

UNIT–IV

8. Using J–operator method, explain parallel resonant circuit. Why it is called rejector circuit? 8

9. (a) Find the current in a circuit having a capacitor of $5\mu F$ and a resistance of 10Ω, when connected to 200V, 50 cycles A.C. supply. 4

(b) Determine the impedance and phase difference between the current and emf in an a.c. circuit containing resistance and inductance using J–operator. 4